A mechanical analog of quantum bradyons and tachyons

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We present a mechanical analog of a quantum wave-particle duality: a vibrating string threaded through a freely moving bead or 'masslet'. For small string amplitudes, the particle movement is governed by a set of non-linear dynamical equations that couple the wave field to the masslet dynamics. Under specific conditions, the particle achieves a regime of *transparency* in which the field and the particle's dynamics appear decoupled. In that special case, the particle conserves its momentum and a guiding wave obeying a Klein-Gordon equation, with real or imaginary mass, emerges. Similar to the double-solution theory of de Broglie, this guiding wave is locked in phase with a modulating group-wave co-moving with the particle. Interestingly, both subsonic and supersonic particles can fall into a quantum regime as with the slower-than-light bradyons and hypothetical, faster-than-light tachyons of particle physics.

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I. INTRODUCTION

The foundations of Quantum mechanics (QM) mainly rely on the pioneering work of Louis de Broglie elaborated during his PhD [1] and for which he received the Nobel prize. The key assumption of de Broglie's intuitive approach is that any mass m_p of matter acts like a clock of pulsation ω_p such that its mass energy $m_\text{p}c^2$ balances its vibrational energy $\hbar\omega_\text{p}$ where c is the light velocity and $\hbar$ the Planck constant. Then, relying on special relativity to estimate how this moving clock would appear for an immobile observer, de Broglie showed that the clock must be in phase with a superluminal phase wave, the guiding wave, giving birth to the celebrated but still mysterious wave-particle duality. After de Broglie discovered this phase wave, he proposed in late 1926 a mechanical analog [2–4], 'the double solution theory'. This analog was subsequently followed by some hydrodynamical and very interesting analogs *e.g.* the Madelung approach [5]. In the same series of works [3, 6], de Broglie also introduced the pilot-wave interpretation nowadays known as de Broglie-Bohm – or 'Bohmian' mechanics – after its rediscovery in 1952 by D. Bohm [7, 8]. While the quest for a classical analog of QM is legitimate, it is far from being an easy task and de Broglie failed in extending his earlier results. Quite recently, the interest on this subject was renewed by the pioneering work initiated by Couder and Fort on bouncing droplets [9–11] in which one or several droplets hit a vertically shaken bath and generate a surface wave. Among other works (see [12] for a review), these droplets, sometimes referred to as walkers, were shown to not only mimic a wave particle duality at macroscale, but also to reproduce most, if not all, of QM features. This is not that surprising because these models share some features with the double solution and pilot wave theories [3, 7], known to be possible alternative interpretations, however deterministic, of quantum mechanics.

In this paper, we focus on a type of mechanical analog closer in spirit to the original double solution of de Broglie, i.e., transverse waves on which a small bead of mass m_p and of stiffness k_p is submitted to the string impulse and moves without friction. This 'masslet' acts as a particle somewhat similar to a free-to-move defect or impedance jump; the density and elasticity are locally altered at the particle location. Its dynamical behavior is governed by usual momentum transfer at the impedance jump, giving rise to reflection, transmission or absorption of the incoming wave. The masslet being free to move, these mechanisms are accompanied with the radiation force phenomenon common to acoustic and electromagnetic fields. This drives the particle in the whole field – incident and scattered – and eventually leads to memory effects. In the spirit of de Broglie's assumptions, the sliding masslet can be viewed as a a moving clock of pulsation $\omega_\text{p} = \sqrt{k_\text{p}/m_\text{p}}$ in its rest frame.

In the past, quite similar models were first proposed by Rayleigh and Helmholtz [13–15] to study the vibrations of a loaded string. More recently, Boudaoud *et al.* studied the self-adaptation of free-to-move beads on a string submitted to acoustic noise [16] in the context of soap film and pattern formation. A few years ago, Borghesi showed relying on a relativistic framework, that such a system yields a wave particle duality governed by the also relativistic Klein-Gordon equation [17]. Here, we restrict ourselves to the non-relativistic Newtonian framework and unravel the emergence of a *transparency* regime in which the particle-string interaction, and thus the radiation force, vanish. As we show, this regime, is reminiscent of de Broglie's double-solution [3] and leads to a Schrödinger equation for the phase wave associated to the particle. Very interestingly, two classes of transparent particles are possible candidates: (i) the class of subsonic particles that travel uniformly along the string with a velocity smaller than the speed of sound on the string and (ii) the class of supersonic (i. e., faster-than-

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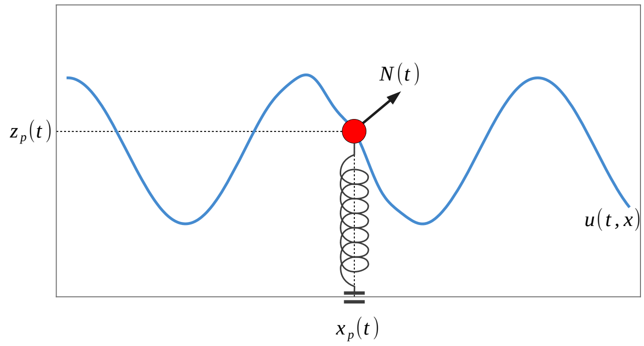

FIG. 1: Sketch of the string-mass mechanical analog. A bead of coordinates $(x_\mathrm{p}, z_\mathrm{p})$ slides without friction on a string with local transverse amplitude $u(t, x)$. A vertical spring acts on the bead as as a vertical restoring force and features an internal clock in the de Broglie double solution of quantum mechanics.

sound) particles. These two families of dynamical motions are hereafter referred to as bradyons and tachyons, respectively in reference to their quantum counterparts in particle physics (where the speed of sound must be replaced by the speed of light). The paper is structured as follows: In Section. II, we describe the system and derive the general dynamical equations. We discuss analytical solutions and present the numerical approach employed for studying the dynamics. In Section. III, we discuss in detail the transparency regimes by focusing on the two quantum analogs of bradyons and tachyons. Concluding remarks are addressed in Section. IV.

II. THE STRING-MASSLET SYSTEM

A. General description

Our system consists of an elastic string of linear density λ stretched along x by tension T. The string oscillates in the transverse z-direction and the vibration is characterized by the field $u(t, x)$. The effect of gravity is neglected throughout the paper. A ring-shaped mass m_p – or bead – is threaded on this string at $x = x_\mathrm{p}$ and can slide without friction.

Importantly, in order to make this particle behave as a clock, the masslet is placed in an additional elastic force field corresponding to a spring of stiffness k_p. Thus, it exerts an elastic force towards the non-deformed axis of the string ($z = 0$) (see illustration in Fig. 1).

Since the mass cannot escape the string, its vertical position z_p must satisfy the holonomic condition $z_\mathrm{p}(t) = u(t, x_\mathrm{p}(t))$ at any time t. Furthermore, the absence of any dissipation (and in particular of friction between the particle and the string) imposes that the reaction force $N(t)$ from the string to the particle is locally normal to the string at the particle's location (in the limit of small vibration amplitudes *i.e* the reaction force is mainly vertical). When the string vibrates, the mass is accelerated vertically and horizontally by the local string acceleration and subsequently moves along it. Since the particle has locally a density or elasticity which are different from the string, its inertia will in turn stretch the string and act as a source for the field $u(t, x)$, generating scattered waves. Thereby, the particle acts as a clock moved by and within the wave field that it has formerly produced.

In this work, we only study non-relativistic movements, *i.e.* we assume that the velocity of particle and wave are much smaller than the celerity of light defined by the velocity c of the elastic waves along the string. This allows us to define two distinct regimes associated to the Mach number $Ma = |v_{\mathrm{p},x}/c|$ where $v_{\mathrm{p},x}$ is the horizontal velocity of the particle. $Ma < 1$ is the subsonic regime, while $Ma > 1$ is the supersonic one. As we show in the following, the subsonic regime is physically equivalent to the bradyonic regime as defined in particle physics for slower-than-light quanta, while the supersonic regime is similar to the tachyonic one with faster than light quantum particles. The mechanical analogy with special relativity is remarkable and we demonstrate that the application of an equivalent Lorentz transformation (where the sound velocity replaces the velocity of light) is crucial for a quantum description of mechanical analogs.

B. Deriving the equations

Since our system possesses several degrees of freedom coupled by a holonomic condition, the Lagrangian formalism is well suited to derive the equations of motion. Obviously, the total action I of the system can be split in three parts as

$$I = \int dt L_\mathrm{p} + \iint dt dx \mathcal{L}_\mathrm{s} + \int dt L_{\mathrm{int.}} \tag{1}$$

where L_p and $\mathcal{L}_\mathrm{s}$ are respectively the Lagrangian of the particle and the Lagrangian density of the string (indicated by the cursive letter) in the absence of the holonomic coupling constraint $L_{\mathrm{int.}}$. We focus first on the expression of the Lagrangian L_p of the particle in our system. Since we impose an attraction towards the string baseline, a potential

$$V(z_\mathrm{p}) = \frac{1}{2} m_\mathrm{p} \omega_\mathrm{p}^2 z_\mathrm{p}^2 \tag{2}$$

is introduced, with ω_p a pulsation or equivalently $k_\mathrm{p} = m_\mathrm{p}\omega_\mathrm{p}^2$ a stiffness applied to the particle solely. The particle Lagrangian then takes the form

$$L_\mathrm{p}(t, z_\mathrm{p}, v_\mathrm{p}) = \frac{1}{2} m_\mathrm{p} v_\mathrm{p}^2 - V(z_\mathrm{p}) \tag{3}$$

where we have introduced $\vec{v}_p = v_{p,x}\vec{u}_x + v_{p,z}\vec{u}_z$ with $v_{\mathrm{p},x}(t) = \frac{dx_\mathrm{p}(t)}{dt}$ and $v_{\mathrm{p},z}(t) = \frac{dz_\mathrm{p}(t)}{dt}$ the longitudinal and transverse particle velocity respectively. Without the holonomic constraint, the variational Lagrange principle $\delta[\int dt L_\mathrm{p}] = 0$ leads to the usual Newtonian equations

$$m_\mathrm{p}\ddot{x}_\mathrm{p}(t) = 0,\ \ m_\mathrm{p}[\ddot{z}_\mathrm{p}(t) + \omega_\mathrm{p}^2 z_\mathrm{p}(t)] = 0. \tag{4}$$

Similarly, $\mathcal{L}_\mathrm{s}$ is obtained by taking the continuum limit of a chain of springs with constant stiffness and considering small amplitude vibrations (i.e., neglecting nonlinearities):

$$\mathcal{L}_\mathrm{s}(u, \partial_t u, \partial_x u) = \frac{1}{2}\lambda(\partial_t u(t,x))^2 - \frac{1}{2}T(\partial_x u(t,x))^2 \tag{5}$$

In this equation, λ and T are respectively the linear density and the tension of the string. We also define our local speed of sound $c = \sqrt{T/\lambda}$ of the transverse waves propagating along the string. Once again, for $L_\mathrm{int.} = 0$, one can obtain from the variational principle $\delta[\int dt \int dx \mathcal{L}_\mathrm{s}] = 0$ the wave-equation:

$$\lambda\partial_t^2 u(t,x) - T\partial_x^2 u(t,x) = \Box u(t,x) = 0 \tag{6}$$

where $\Box$ is the usual linear d'Alembert operator.

The specific nature of the holonomic constraint is ideally grasped by the coupling Lagrangian:

$$L_\mathrm{int.}(t, x_\mathrm{p}, z_\mathrm{p}, N) = N \cdot [z_\mathrm{p} - u(t, x_\mathrm{p})], \tag{7}$$

or equivalently by a Lagrangian density $\mathcal{L}_\mathrm{int.}$ (such that $L_\mathrm{int.} = \int dx \mathcal{L}_\mathrm{int.}$):

$$\mathcal{L}_\mathrm{int.}(u, x_\mathrm{p}, z_\mathrm{p}, N) = N \cdot [z_\mathrm{p} - u(t,x)]\delta(x - x_\mathrm{p}), \tag{8}$$

where $N(t)$ is a Lagrange multiplier defining an additional variable in the variational problem. Variations with respect to N lead automatically to the holonomic constraint:

$$z_\mathrm{p} = u(t, x_\mathrm{p}). \tag{9}$$

Taking into account the action of the mass on the string, the wave dynamics follows a new equation obtained from the variational principle $\delta[\int dt \int dx(\mathcal{L}_\mathrm{s} + \mathcal{L}_\mathrm{int.})] = 0$, whereas symmetrically the modified dynamic of the sliding mass is obtained from $\delta[\int dt(L_\mathrm{p} + L_\mathrm{int.})] = 0$. Altogether this leads to the following set of coupled equations:

$$m_\mathrm{p}\ddot{x}_\mathrm{p}(t) = -\partial_x u(t,x)|_{x=x_\mathrm{p}(t)} N(t), \tag{10a}$$

$$N(t) = m_\mathrm{p}\left[\ddot{z}_\mathrm{p}(t) + \omega_\mathrm{p}^2 z_\mathrm{p}(t)\right], \tag{10b}$$

$$\Box u(t,x) = -\frac{N(t)}{T}\delta(x - x_\mathrm{p}(t)) \tag{10c}$$

with the holonomic condition Eq. 9. Interestingly, after elimination of $N(t)$ Eqs. 10a and 10b can equivalently be rewritten as

$$\ddot{x}_\mathrm{p}(t) = -\partial_x u(t,x)|_{x=x_\mathrm{p}(t)}\left[\ddot{z}_\mathrm{p}(t) + \omega_\mathrm{p}^2 z_\mathrm{p}(t)\right] \tag{11}$$

Remarkably, this equation is independent of the mass m_p a fact which is reminiscent of acoustic analogs of gravitational forces in hydrodynamical systems [19]. We also point-out that Eqs. 10a, 10b, and 10c can easily be interpreted using a Newtonian language. The Lagrange multiplier $N(t)$ is the vertical reaction force acting on the sliding mass while $-\partial_x u(t,x)|_{x=x_\mathrm{p}(t)} N(t)$ is the horizontal component of this reaction force (i.e., in the linear limit of small wave-amplitude we have $\partial_x u(t,x)|_{x=x_\mathrm{p}(t)} = \tan\theta \simeq \sin\theta$ where θ is the angle between the reaction force N and the z vertical direction). We emphasize that Eq. 10c can be rewritten as $\tilde{\lambda}(t,x)\partial_t^2 u(t,x) = T\partial_x^2 u(t,x) = 0$ with $\tilde{\lambda}(t,x) = \lambda + m\delta(x - x_\mathrm{p}(t))$. In other words the mass m can also be interpreted as a local translatable defect in the linear density λ. This could have an impact for physical interpretations. Moreover, the previous dynamics is completed by an analysis of energy and momentum conservation in the coupled system (*i.e.*, of prime integrals of motion). From the full action and Lagrangian Eq. 1 (or alternatively from Eqs. 10a, 10b, and 10c) we deduce after lengthy but straightforward calculations the local energy-momentum conservation laws for the field coupled to the mass:

$$\begin{aligned}\partial_t\varepsilon(t,x) + \partial_x S_x(t,x) &= -N\delta(x - x_\mathrm{p}(t)\partial_t u(t,x)|_{x=x_\mathrm{p}(t)}\\ \partial_t g_x(t,x) + \partial_x T_{xx}(t,x) &= N\delta(x - x_\mathrm{p}(t)\partial_x u(t,x)|_{x=x_\mathrm{p}(t)}\end{aligned} \tag{12}$$

where $\varepsilon = \frac{1}{2}T[\frac{1}{c^2}(\partial_t u)^2 + (\partial_x u)^2]$ and $g_x = -\frac{T}{c^2}\partial_t u \partial_x u$ are respectively the $u-$field energy and linear (pseudo) momentum density of the acoustic field along the string. Similarly $S_x = c^2 g_x$ and $T_{xx} = \varepsilon$ are respectively the energy and (pseudo) momentum density flow along the x direction (T_{xx} is the constraint tensor of the field which has only one component here). Moreover, combining Eq. 12 with Eqs. 10a, 10b, and 10c leads to

$$\begin{aligned}\frac{d}{dt}\left\{\int dx\varepsilon(t,x) + \frac{1}{2}m_\mathrm{p}(\dot{x}_\mathrm{p})^2\right.\\ \left. + \frac{1}{2}m_\mathrm{p}\left[(\dot{z}_\mathrm{p})^2 + \omega_\mathrm{p}^2(z_\mathrm{p})^2\right]\right\} = 0\end{aligned} \tag{13a}$$

$$\frac{d}{dt}\left\{\int dx g_x(t,x) + m_\mathrm{p}\dot{x}_\mathrm{p}\right\} = 0 \tag{13b}$$

which shows that the total energy and linear momentum of the system are conserved.

We stress that despite its non-relativistic nature our model differs from the one proposed in [17] which considered a Klein-Gordon wave equation from the start, *i.e.*, $[\Box + \frac{\Omega_m^2}{c^2}]u(t,x) = -\frac{N(t)}{T}\delta(x - x_\mathrm{p}(t))$. In [17] it was shown that such a Klein-Gordon equation with a source term can be used to generate a mechanical analog of wave-particle duality. Here we show that it is not necessary to introduce such a complication. An inhomogeneous d'Alembert equation, *i.e.*, as in Eq. 10c, is already sufficient to reproduce wave-particle duality. As we show in the following this is in complete agreement with original ideas developed by de Broglie before 1927 [18].

III. THE TRANSPARENCY REGIME

A. The bradyonic or subsonic regime: wave-particle duality

Let us now focus on a particular class of motion for which the coupling force $N(t)$ between the field and the particle cancels:

$$m_\mathrm{p}[\ddot{z}_\mathrm{p}(t)+\omega_\mathrm{p}^2 z_\mathrm{p}(t)]=N(t)=0 \quad \text{(transparency)} \tag{14}$$

yielding a purely oscillatory motion

$$z_\mathrm{p}(t)=A\cos\left(\omega_\mathrm{p}t+\varphi\right) \tag{15}$$

with A and φ real constants. Eqs. 10a and 10c yield:

$$\ddot{x}_\mathrm{p}(t)=0, \quad \Box u(t,x)=0 \tag{16}$$

which results in an inertial movement for the particle:

$$x_\mathrm{p}(t)=v_\mathrm{p}t+x_\mathrm{p,i} \tag{17}$$

with $x_\mathrm{p}(0)=x_\mathrm{p,i}$ and $v_\mathrm{p}(t)=v_\mathrm{p}$ two real constants. In this configuration the different degrees of freedom are decoupled and in particular $\int dx\varepsilon(t,x)$, $\frac{1}{2}m_\mathrm{p}(\dot{x}_\mathrm{p})^2$, $\frac{1}{2}m_\mathrm{p}\left[(\dot{z}_\mathrm{p})^2+\omega_\mathrm{p}^2(z_\mathrm{p})^2\right]$, $\int dx g_x(t,x)$ and $m_\mathrm{p}\dot{x}_\mathrm{p}$ are constant of motions. We emphasize that the oscillatory motion $z_\mathrm{p}(t)$ is reminiscent of de Broglie's original idea [1, 18] of associating a local clock to any particle. Here, we have a more detailed mechanical model for which we see that the uniform motion given by Eq. 17 is dynamically linked to the harmonic oscillation defined by Eq. 15.

Concerning the string field $u(t,x)$, several solutions are *a priori* available. Indeed the general solution of $\Box u(t,x)=0$ reads $u(t,x)=f(t-x/c)+g(t+x/c)$ corresponding to two arbitrary pulses $f(t)$ and $g(t)$ propagating respectively along the $+x$ and $-x$ direction. Considering for example the case $g=0$ and using the holonomic condition Eq. 9 one gets $f(t(1-\frac{v_\mathrm{p}}{c})-\frac{x_\mathrm{p,i}}{c})=A\cos\left(\omega_\mathrm{p}t+\varphi\right)$, i.e.,

$$f(t)=A\cos\left[\frac{\omega_\mathrm{p}}{1-\frac{v_\mathrm{p}}{c}}\left(t+\frac{x_\mathrm{p,i}}{c}\right)+\varphi\right]. \tag{18}$$

This motion would correspond to a mass 'surfing' on a monochromatic propagative wave with pulsation $\frac{\omega_\mathrm{p}}{1-\frac{v_\mathrm{p}}{c}}$. However, while Eq. 18 is interesting in itself we are not going here to further develop this approach. Instead, we now follow the physical intuitions of de Broglie and seek solutions for the homogeneous equation $\Box u=0$ such that in a co-moving inertial frame $\mathcal{R}'$ translating at velocity v_p with the particle, the field u would appear as stationary. Interestingly, a Galilean transformation of the coordinates ($t'=t$, $x'=x-v_\mathrm{p}t$) which lets the time flow unchanged fails to bring a standing solution. This coordinate change, although legitimate here for a Newtonian approach (small Mach number) is however too crude to allow for clock synchronization at a distance. A coordinate transformation that does not let the time unaltered is required to bring a stationary solution. Actually, a 'first-order' Poincaré-Lorentz transformation of the form: $x'=x-v_\mathrm{p}t$, $t'=t-\frac{v_\mathrm{p}}{c^2}x$ is already sufficient to ensure the invariance of the free wave equation. This is because the term $-\frac{v_\mathrm{p}}{c^2}x$ results from a time synchronization procedure for clocks located at different points of inertial frames $\mathcal{R}$ and $\mathcal{R}'$ as already proposed by Poincaré in 1900 [20]. Moreover, the d'Alembert operator $\Box$ being invariant under the usual Lorentz transformation $\Box=\Box'$, we make use of the following coordinate change:

$$x'=\gamma_\mathrm{p}\left(x-v_\mathrm{p}t\right), \quad t'=\gamma_\mathrm{p}\left(t-\frac{v_\mathrm{p}}{c^2}x\right), \tag{19}$$

with $\gamma_\mathrm{p}^{-1}=\sqrt{1-\frac{v_\mathrm{p}^2}{c^2}}$ so that in the Lorenz-Poincaré group in dimension 1+1, the field $u(t,x)$ appears as a scalar invariant field, *i.e.*, $u(t,x)=u'(t',x')$. We point out that the variable t' and x' have no direct physical meaning here since we are working in the context of Newtonian dynamics where the time is absolute. However, Eq. 19 is used as a mathematical tool for finding the solutions of the d'Alembert equation under the restriction $Ma<1$ (in this context we emphasize that Voigt [21] already used the Lorentz transformation as a mathematical tool in optics but devoid of physical interpretation). Searching for a standing solution in the Lorentzian comoving frame $\mathcal{R}'$ that ensures a 'time-space' separation $u'(t',x')=F(t')G(x')$, one gets:

$$\frac{1}{c^2}\frac{d^2F}{dt'^2}G-F\frac{d^2G}{dx'^2}=0 \tag{20}$$

which turns into a set of equation:

$$\frac{d^2F}{dt'^2}+\omega'^2F=0, \quad \frac{d^2G}{dx'^2}+\frac{\omega'^2}{c^2}G=0 \tag{21}$$

with ω' a complex constant to be determined. For the case of interest when ω' is real, F and G are harmonic so that we obtain an 'amplitude modulated' field $u'(t',x')$:

$$\begin{aligned} u'(t',x') &= B\cos\left(\omega't'+\eta\right)\cos\left(\frac{\omega'}{c}x'+\xi\right)=u(t,x) \\ &= B\cos\left[\omega'\gamma_\mathrm{p}\left(t-\frac{v_\mathrm{p}}{c^2}x\right)+\eta\right]\cos\left[\frac{\omega'}{c}\gamma_\mathrm{p}\left(x-v_\mathrm{p}t\right)+\xi\right] \end{aligned} \tag{22}$$

B, η and ξ being three real constants. They can be determined by using the holonomic condition expressed for the case of the uniform motion: $z_\mathrm{p}(t)=u(t,x=v_\mathrm{p}t+x_\mathrm{p,i})$. It follows that:

$$\begin{aligned} u(t,x=vt+x_\mathrm{p,i}) = B\ &\cos\left(\frac{\omega'}{\gamma_\mathrm{p}}t+\eta-\frac{\omega'\gamma_\mathrm{p}x_\mathrm{p,i}v_\mathrm{p}}{c^2}\right) \\ \times\ &\cos\left(\frac{\omega'}{c}\gamma_\mathrm{p}x_\mathrm{p,i}+\xi\right) \end{aligned}$$

which, by identification with $z_0(t)$ given by Eq. 15 yields:

$$\varphi=\eta-\frac{\omega'\gamma_\mathrm{p}x_\mathrm{p,i}v_\mathrm{p}}{c^2}+2\pi n \quad (\text{with } n\in\mathbb{Z}), \tag{23}$$

$$A = B\cos\left(\frac{\omega'}{c}\gamma_{\rm p}x_{\rm p,i}+\xi\right), \tag{24}$$

and above all the relation:

$$\omega_{\rm p} = \frac{\omega'}{\gamma_{\rm p}}. \tag{25}$$

To give a meaningful interpretation of this condition, we can write: $u' = u$ in Eq. 22 in the form $u = B\cos S_{\rm brad.}\cos\Phi_{\rm brad.}$ with

$$S_{\rm brad.} = \omega' t' + \eta = \omega t - kx + \eta$$
$$\Phi_{\rm brad.} = \frac{\omega'}{c}x' + \xi = \frac{\omega}{c}(x - v_{\rm p}t) + \xi \tag{26}$$

with $\omega = \omega'\gamma_{\rm p}$, $k = \omega v_{\rm p}/c^2$. Besides, the following dispersion relation applies

$$\frac{\omega^2}{c^2} - k^2 = \frac{\omega'^2}{c^2} \tag{27}$$

between the pulsation ω and the wave-vector k. The quantity $S_{\rm brad.}$ plays the role of a phase for a plane (carrying) wave, $\psi(t,x) = e^{iS_{\rm brad.}}$ solution of the Klein-Gordon equation:

$$\left(\frac{1}{c^2}\partial_t^2 - \partial_x^2\right)\psi(t,x) = -\frac{\omega'^2}{c^2}\psi(t,x). \tag{28}$$

The phase velocity $v_{\rm ph.}$ is defined by the condition $dS_{\rm brad.} = 0$, which implies

$$v_{\rm ph.} = \frac{\omega}{k} = \frac{c^2}{v_{\rm p}} > c \tag{29}$$

in accordance with the formulas obtained by Louis de Broglie in his PhD manuscript [2]. Besides, $\Phi_{\rm brad.}$ in Eq. 26 defines an envelop (*i.e.*, a group) velocity which identifies with the particle's velocity $v_{\rm p}$ (setting $d\Phi_{\rm brad.} = 0$ we get $dx/dt = v_{\rm p}$). Furthermore, we deduce the Rayleigh formula $v_{\rm gr.} = \frac{d\omega}{dk} = v_{\rm p}$ which was also obtained by de Broglie. This is clearly reminiscent of Hamilton formula $\frac{dH_{\rm p}}{dP_{\rm p}} = v_{\rm p}$ if we identify the Hamiltonian or energy function $H_{\rm p} := m_{\rm p}c^2\gamma_{\rm p}$ and the linear momentum $P_{\rm p} := m_{\rm p}\gamma_{\rm p}v_{\rm p}$ with respectively $Q\omega$ and Qk where Q is a constant having the dimension of an action. The similarity between Q and $\hbar$ is clear and is reenforced if we write Eq. 28 in the limit $M_a \ll 1$ as

$$iQ\partial_t\Psi \simeq -\frac{Q^2}{2m_{\rm p}}\partial_x^2\psi(t,x) + m_{\rm p}c^2\psi(t,x) \tag{30}$$

which is identical to Schrödinger's equation after the substitution $Q \to \hbar$ (similarly in the Klein-Gordon Eq. 28 we can replace $\frac{\omega'^2}{c^2}$ by $\frac{m_{\rm p}^2c^2}{Q^2}$ in agreement with standard textbooks). Moreover, we can also write $H_{\rm p} = -Q\partial_t S_{\rm brad.}$ and $P_{\rm p} = Q\partial_x S_{\rm brad.}$ which are reminiscent of Hamilton-Jacobi equations with QS playing the role of an action and therefore we have

$$v_{\rm p} = -c^2\frac{\partial_x S_{\rm brad.}}{\partial_t S_{\rm brad.}} \tag{31}$$

which is the guidance formula introduced by de Broglie in his pilot-wave interpretation [3, 6] and which leads to $v_{\rm p} = Q\frac{\partial_x S_{\rm brad.}}{m_{\rm p}}$ in the limit $Ma \ll 1$ in agreement with Bohmian mechanics [6, 7]. Therefore, altogether we recover de Broglie's assumptions casting the double solution theory [3] with a ψ-wave also called the guiding wave (solution of the Klein-Gordon equation) and the u-wave (solution of the homogeneous d'Alembert equation) associated with the particle's movement. Importantly, in our approach (and in contradistinction to [17]) we start from the d'Alembert equation and not from the Klein-Gordon equation. Still, we are able to obtain a guiding-wave ψ which is solution of Eq. 28 *i.e.*, the Klein-Gordon equation. In other words the mass term of the Klein-Gordon equation has been generated from the $u-$field itself. This agrees with a model already presented by de Broglie in 1925 [18], *i.e.*, two years before the model named traditionally the 'double solution'[3] and based on the Klein-Gordon equation.

The condition (25) on the frequency can also be written in terms of phase and becomes a phase-locking condition, since for $x = x_{\rm p}(t)$ one gets:

$$S_{\rm brad.} = \omega' t' + \eta = \frac{\omega'}{\gamma_{\rm p}}t + \varphi = \omega_{\rm p}t + \varphi \tag{32}$$

which expresses the phase-locking of the particle's clock (with pulsation $\omega_{\rm p}$) to that of the wave (with pulsation ω or ω'). Following de Broglie [18, 22], we can rewrite the total wave u as a sum of waves (by means of the trigonometric identity: $2\cos F\cos G = \cos(F+G) + \cos(F-G)$) to give a physically meaningful interpretation

$$\begin{aligned} u'(t',x') = &\frac{B}{2}\cos\left[\omega'\left(t' + \frac{x'}{c}\right) + \eta + \xi\right] \\ &+\frac{B}{2}\cos\left[\omega'\left(t' - \frac{x'}{c}\right) + \eta - \xi\right] \end{aligned} \tag{33}$$

or equivalently

$$\begin{aligned} u(t,x) \;=\; &\frac{B}{2}\cos\left[\omega'\gamma_{\rm p}(1-\frac{v_{\rm p}}{c})(t+\frac{x}{c}) + \eta + \xi\right] \\ +\; &\frac{B}{2}\cos\left[\omega'\gamma_{\rm p}(1+\frac{v_{\rm p}}{c})(t-\frac{x}{c}) + \eta - \xi\right] \end{aligned} \tag{34}$$

It is another way to express u as the sum of two counter propagating waves $u(t,x) = u_+(t,x) + u_-(t,x)$ with (i) $u_- = \frac{B}{2}\cos[\omega_- t + \omega_- x/c + \eta + \xi]$ a wave with a low-frequency Doppler shift $\omega_- = \omega(1 - \frac{v_{\rm p}}{c})$ propagating along the $-x$ direction, and (ii) $u_+ = \frac{B}{2}\cos[\omega_+ t - \omega_+ x/c + \eta - \xi]$ a wave with a high-frequency Doppler shift $\omega_+ = \omega(1 + \frac{v_{\rm p}}{c})$ propagating along the $+x$ direction. Experimentally, this decomposition will allow us to generate the resulting modulated u wave appearing in Eq. 22 as the sum of two plane waves.

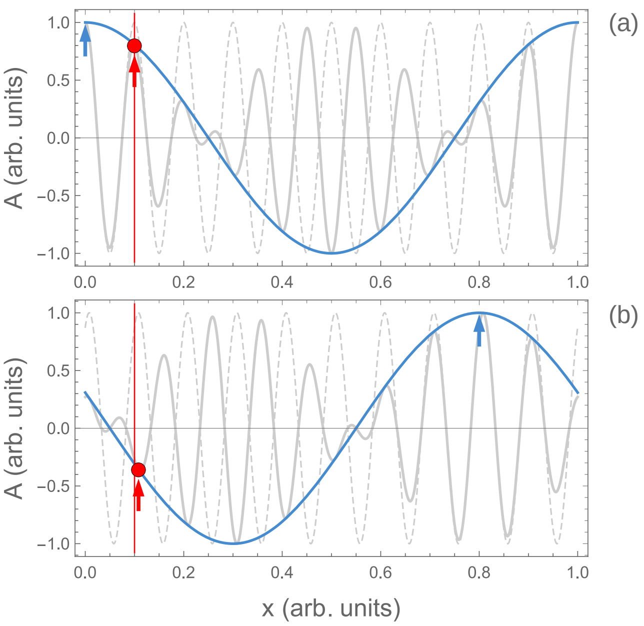

FIG. 2: The transparent regime for a bradyonic (*i.e.*, subsonic) particle plotted from the analytical solutions described in the paper. An equivalent animation obtained from the numerical code of the full set of equation is available in the supplemental material [23]. For this example, we have $v_{\mathrm{p}}/c = 0.1$, $x_{\mathrm{p,i}} = 0.1$ for a string length of $L = 1$ and $c = 1$ (in natural units). We have chosen a $\psi-$wave temporal period $T_{\mathrm{phase}} = \frac{2\pi}{\omega} = 10$ and $\eta = \xi = 0$. The continuous thick gray line is the total field $u(x,t)$ which can be decomposed into the phase-wave ψ (continuous blue line) and the group wave (dashed gray line). The whole field is shown at two different instants (a) $t = 0$ and (b) $t = 0.08$ clearly demonstrating the subsonic (or bradyonic) regime of the particle. The particle's initial condition is set so that it is at a maximum of the group phase $\Phi_{\mathrm{brad.}}$ (see Eq. 26) and is clearly locked to it along time. The red arrows in panel (a) and (b) compare the position of the particle at time $t = 0$ and $t = 0.08$ whereas the blue arrow show the position and displacement of a phase maximum at the two instants $t = 0$ and $t = 0.08$.

The analytical solution is plotted in Fig. 2 for this transparency regime (the normalized parameters are indicated in the caption). Going back to Eq. 26 we have for the phase-wave a temporal period $T_{\mathrm{phase}} = \frac{2\pi}{\omega}$ and a spatial period $\lambda_{\mathrm{phase}} = \frac{2\pi}{\omega}\frac{c^2}{v_{\mathrm{p}}}$ which have to be compared with the temporal and spatial period of the group wave $T_{\mathrm{group}} = \frac{2\pi}{\omega}\frac{c}{v_{\mathrm{p}}} \geq T_{\mathrm{phase}}$, $\lambda_{\mathrm{group}} = \frac{2\pi}{\omega}c \leq \lambda_{\mathrm{phase}}$ (as illustrated in Fig. 2 for $\lambda_{\mathrm{phase}}/\lambda_{\mathrm{group}} = 10$. Importantly, we have here the well-known de Broglie formula [1]:

$$P_{\mathrm{p}} = \frac{2\pi Q}{\lambda_{\mathrm{phase}}} \tag{35}$$

It is interesting to note that in de Broglie approach the wave field was a solution of a non-homogeneous equation including a source term for the particle $\Box u(t,x) \propto \delta(x - x_{\mathrm{p}}(t))$ (i.e., like in Eq. 10c). Here, instead the transparency condition $N(t) = 0$ imposes the field equation $\Box u(t,x) = 0$ and this simplifies the interpretation of Eq. 34 as the sum of two plane waves whereas de Broglie had to involve a complicated sum of advanced and retarded waves emitted by the particle itself [18, 22].

More generally, it is clear that the equations of motion presented here are strongly non-linear, and except for a few specific regimes as the ones discussed in the present paper, no analytical solutions are usually reachable. In order to further investigate the physics of our model, we have employed a numerical scheme based on standard finite differences through a Runge-Kutta of order 4 (see for example [24]). Since the mutual interaction between the mass and the string only enters as an external source term for the each other, we have implemented separately their differential equations and solve the full system self consistently. We first start by fixing initial compatible configurations for both the string and the mass. During the time resolution process, an update of the string is performed, accounting the presence of the mass. Once this space-time update is achieved, we proceed to the update of the mass now accounting back to the new state of the string. And we continue so on and so forth until desired time periods. The convergence is ensured by respecting the von Neumann stability criterion. Despite the above mentioned non-linearities in the complete set of equations, the algorithm appears to be well stable and reproduces very precisely the analytical solutions discussed here. (transparency). This validates our approach and indicates that the coupling between the mass and the string does not seem to be that crucial in the convergence. For this first work, we have employed our algorithm in order to complement with the dynamics the discussed analytical solutions. In particular, we have recorded a movie [23] showing the behavior of the mass in the transparency regime. The question of other possible emerging exotic regimes in our coupled system is very interesting though, and deserves a proper study. We dedicate a more systematic numerical analysis to a future work.

The calculations above deserve some comments in relation with de Broglie's picture. Indeed, it is noteworthy that the pulsation ω_{p} remains unchanged regardless of the particle's speed and is in particular different from ω' (which is velocity dependent). It is slightly different from de Broglie's results and is a consequence of our mixed approach, combining a Lorenz transformation (by means of c the speed of sound) in a Newtonian framework for which relativistic dynamics do not apply. More precisely, in de Broglie's approach which is fully relativistic (*i.e.*, with the sound celerity replaced everywhere by the light velocity) the particle's internal clock with pulsation ω_{p} is defined in the rest-frame $\mathcal{R}'$, not in the laboratory frame $\mathcal{R}$. The phase-locking condition reads now:

$$S_{\mathrm{brad.}} = \omega' t' + \eta = \frac{\omega'}{\gamma_{\mathrm{p}}}t + \eta - \frac{\omega'\gamma_{\mathrm{p}}x_{\mathrm{p,i}}v_{\mathrm{p}}}{c^2} = \omega_{\mathrm{p}}t' + \varphi \tag{36}$$

and we have thus $\omega' = \omega_{\mathrm{p}} = \frac{\omega}{\gamma_{\mathrm{p}}}$ which differs from Eq. 25 by a prefactor γ_{p} associated with a relativistic time-dilation (we have also $\eta = \varphi$ since φ is now defined in the rest frame[17, 18]). In the non-relativistic limit where $\frac{v_\mathrm{p}}{c} \ll 1$ and $\gamma_\mathrm{p} \simeq 1$ de Broglie's theory reduces to $\omega \simeq \omega' = \omega_\mathrm{p}$. This is identical in our model to the regime $Ma \ll 1$ (where c is now the sound velocity) so that the difference between the two approaches vanishes for sufficiently slow particle motions. It is interesting to remark that in the $Ma \ll 1$ regime the Hamiltonian $H_\mathrm{p} := m_\mathrm{p} c^2 \gamma_\mathrm{p}$ which was formally introduced reduces to $H_\mathrm{p} \simeq m_\mathrm{p} c^2 + \frac{1}{2} m_\mathrm{p} v_\mathrm{p}^2$ which, up to an additive constant, is the translational kinetic energy associated with the particle motion along x (similarly $P_\mathrm{p} \simeq m_\mathrm{p} v_\mathrm{p}$ which identifies with the translational linear momentum of the particle). Since c is here the sound velocity, $m_\mathrm{p} c^2$ can not physically be identified with a rest energy which is a relativistic Einsteinian concept. This once again stresses the similarities and differences between our mechanical analog and de Broglie's own approach. For similar reasons we have no right to identify the integration factor Q discussed previously with the Planck constant $\hbar$. This could only hold in de Broglie's model for a genuine quantum particle. Still, the mechanical analogy works fine for $Ma \ll 1$ and could be actually extended to the relativistic regime by implementing a covariant, *i.e.* Einsteinian, mechanical model. Here, we nevertheless stick to the Newtonian framework which is closer to the experimental realization and is already sufficient to grasp the essential features of de Broglie's mechanical model.

B. The Tachyonic or supersonic regime

So far, we have discussed the dynamics of a particle moving on the string with a velocity v_p smaller than that of the waves on this string. Since this motion is completely governed by the interaction with the field u, we do not expect the particle to spontaneously cross the sound barrier without the use of an external force. However, one could choose initial conditions such that $v_\mathrm{p} > c$. We would then find a whole new supersonic regime, which we can investigate. As explained in the introduction we choose to refer to such a particle as a 'tachyon' by analogy with the eponymous hypothetical particles introduced in special relativity, which are a class of solution for the dynamics associated with faster than light motions [25–28] (symmetrically the case studied previously with $v_\mathrm{p} < c$ is referred to as 'bradyonic' motion [25–28]). Indeed, this velocity c appears in the Lorentz transformation that we use for the field, as in the Lorentz transformation of special relativity, and appears as an asymptotic limit for the velocity v_p in both cases. However one should once again not confuse those two velocities: it is a fundamental limit within the special relativity framework, while it is not in the context of a particle sliding on a string. Supersonic particles (analogous to supersonic aircrafts in a fluid) are indeed possible solutions. Thus, it is completely reasonable to study these 'acoustic' tachyons, which are physically viable solutions of the dynamical equations. For

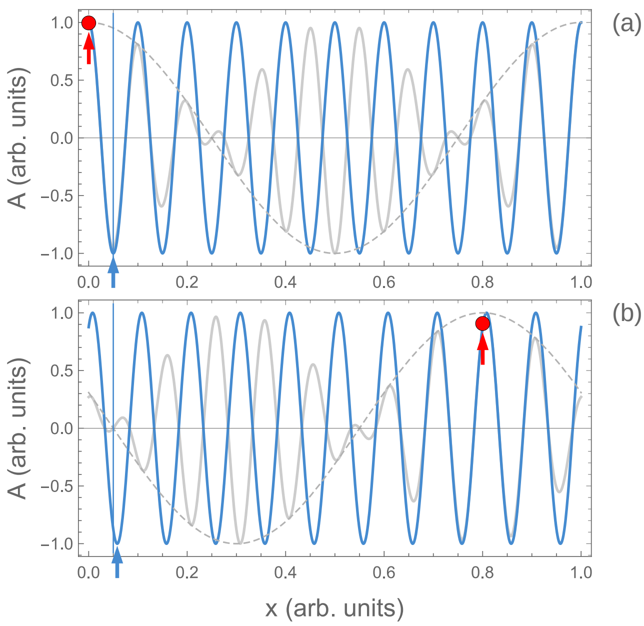

FIG. 3: The transparent regime for a tachyonic (i.e., supersonic) particle of velocity $w_\mathrm{p}/c = 10$, and initial coordinate $X_{\mathrm{p,i}} = 0$ (in natural units). The two panels correspond to observation times (a) $t = 0$ and (b) $t = 0.08$. The other parameters of the wave and string are unchanged with respect to Fig. 2. In particular, we have $T_\mathrm{phase} = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega} \frac{c}{v_\mathrm{p}} = 1$ (see text). Like the subsonic case, the particle is locked to the group wave $\cos \Phi_\mathrm{tach.}$ (dashed gray line) and is clearly faster than the velocity of the phase-wave $\cos S_\mathrm{tach.}$ on the string (blue continuous line). The red and blue arrows compare the displacement of the particle and phase wave between $t = 0$ and $t = 0.08$.

this purpose, we see from Eq. 19 that if we set $x' = 0$, we get $x = v_\mathrm{p} t$ and therefore the ct' axis corresponds to the trajectory of a particle with velocity $v_\mathrm{p} < c$ (this is of course the definition of a comoving rest frame for such a 'bradyonic' motion). If we now set $t' = 0$, we get $x = \frac{c^2}{v_\mathrm{p}} t = w_\mathrm{p} t$ which corresponds to a case where the x' axis identifies with the trajectory of a tachyonic particle with velocity $w_\mathrm{p} = \frac{c^2}{v_\mathrm{p}} > c$. This is the first hint that the dynamics of tachyons is completely symmetrical with that of 'normal' bradyonic particles. Indeed, we can rewrite Eq. 19 as

$$t' = \frac{1}{c} \Gamma_\mathrm{p} (w_\mathrm{p} t - x), \quad x' = c \Gamma_\mathrm{p} \left(\frac{w_\mathrm{p}}{c^2} x - t \right), \qquad (37)$$

with $\Gamma_\mathrm{p} = (\frac{w_\mathrm{p}^2}{c^2} - 1)^{-1/2} = \gamma_\mathrm{p} \frac{v_\mathrm{p}}{c}$ and where we see, by comparing with the original Lorentz transformation, that the roles of space and time have been reversed in the tachyonic case, compared to the bradyonic one. Furthermore, like for Eq. 19 the physical meaning of Eq. 37 is not immediate and here we use it mainly as a mathematical tool for guessing at an interesting solution of the wave equation. More precisely, let us consider once again the stationary field

$$u'(t',x') = B\cos(\omega' t' + \eta)\cos\left(\frac{\omega'}{c}x' + \xi\right) \qquad (38)$$

defined with the variables t' and x'. From the analysis made before we can use this field to match the motion of a tachyonic particle with velocity $w_{\rm p} = \frac{c^2}{v_{\rm p}}$. For this we use in Eq. 38 a wave with the same pulsation ω' as in the badryonic case. However, as we will see below it implies that we use a different spring with pulsation $\Omega_{\rm p} \neq \omega_{\rm p}$. Hence, using Eq. 37 we get in the laboratory frame:

$$u(t,x) = B\cos\left[-\frac{\omega'}{c}\Gamma_{\rm p}(x - w_{\rm p}t) + \eta\right] \times \cos\left[-\omega'\Gamma_{\rm p}\left(t - \frac{w_{\rm p}}{c^2}x\right) + \xi\right]. \qquad (39)$$

We find that, once again, the roles of two quantities have been swapped. We have now

$$\Phi_{\rm tach.} = \omega' t' + \eta = -\frac{\omega'}{c}\Gamma_{\rm p}(x - w_{\rm p}t) + \eta$$
$$S_{\rm tach.} = \frac{\omega'}{c}x' + \xi = -\omega'\Gamma_{\rm p}\left(t - \frac{w_{\rm p}}{c^2}x\right) + \xi \qquad (40)$$

Here, the carrying phase wave and the envelope have been swapped: the supersonic phase wave that we had in the bradyonic case is now traveling alongside the particle; the group wave is now slower than the particle and has become the phase wave (i.e., $S_{\rm tach.} = \Phi_{\rm brad.}$ and $S_{\rm brad.} = \Phi_{\rm tach.}$). The field itself remains unchanged, and only the roles of its two components with respect to the particle have been changed. Thus, all the results that were obtained in the case of a particle with velocity $v_{\rm p}$ can be used for a tachyonic particle of velocity $w_{\rm p} = c^2/v_{\rm p}$ if we keep in mind the symmetrical nature of this supersonic regime. More precisely, considering a uniform motion $x_{\rm p}(t) = w_{\rm p}t + X_{\rm p,i}$ with $X_{\rm p,i}$ a constant and using the holonomic condition Eq. 9 together with the oscillatory $z_{\rm p}(t)$ motion of Eq. 15 we get by identification with Eq. 39:

$$\varphi = -\xi - \frac{\omega'\Gamma_{\rm p}X_{\rm p,i}w_{\rm p}}{c^2} + 2\pi p \quad (\text{with } p \in \mathbb{Z}),$$
$$A = B\cos\left(\frac{\omega'}{c}\Gamma_{\rm p}X_{\rm p,i} - \eta\right),$$
$$\Omega_{\rm p} = \frac{\omega'}{\Gamma_{\rm p}} = \frac{\omega'\frac{c}{v_{\rm p}}}{\gamma_{\rm p}}. \qquad (41)$$

Comparing the frequency-locking condition with the result obtained in Eq. 25 we deduce the constraint

$$\Omega_{\rm p}\frac{c}{v_{\rm p}} = \omega_{\rm p} \qquad (42)$$

and consequently $\Omega_{\rm p} < \omega_{\rm p}$.

There are, however, a few noteworthy differences in some of the equations, mainly the dispersion relation between the wave pulsation $\Omega = \Omega'\Gamma_{\rm p}$ and wave vector $K = \Omega\frac{w_{\rm p}}{c^2}$ which becomes

$$\frac{\Omega^2}{c^2} - K^2 = -\frac{\Omega'^2}{c^2} \qquad (43)$$

and the Klein-Gordon equation for the wave $\Psi = e^{iS_{\rm tach.}}$ (with $S_{\rm tach.} = Kx - \Omega t + \xi$)

$$\Box\Psi = +\frac{\Omega'^2}{c^2}\Psi \qquad (44)$$

where the signs in front of $\frac{\Omega'^2}{c^2}$ have been reversed compared with $\frac{\omega'^2}{c^2}$ in Eq. 27 and Eq. 28. This is specific of tachyonic motions where the pulsation or mass can envisioned as purely imaginary $\tilde{\Omega} = i\Omega'$ (for instance we have $\Omega = \frac{\tilde{\Omega}}{\sqrt{1-\frac{w_{\rm p}^2}{c^2}}}$ [25, 26, 28]). Eq. 44 thus equivalently reads $\Box\Psi = -\frac{\tilde{\Omega}^2}{c^2}\Psi$ which is the usual form for the Klein-Gordon equation but now with a purely imaginary mass.

Once more, we point out that our supersonic particle is only superficially looking as a tachyon. Indeed, genuine relativistic tachyons would induce reluctant causality and thermodynamic problems [25, 26, 28, 29] which are often considered as fatal objections to their mere existence. Here the analogy with tachyon is not complete. For instance observe that we can in analogy with the subsonic regime define the Hamiltonian and linear momentum as $H_{\rm p} := m_{\rm p}c^2\Gamma_{\rm p} = Q\Omega$ and $P_{\rm p} := m_{\rm p}w_{\rm p}\Gamma_{\rm p} = QK$. Physically this corresponds to a tachyonic particle of imaginary mass $im_{\rm p} = iQ\Omega'/c^2 = iQ\omega'/c^2$ while the physical 'Newtonian' mass is of course $m_{\rm p}$. These expressions are in general clearly different from the usual kinetic energy and momentum of a Newtonian particle.

To illustrate the tachyon dynamics we show in Fig. 3 an example for this transparency regime (the normalized parameters are indicated in the caption). Here we have for the phase wave the temporal period $T_{\rm phase} = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega}\frac{c}{v_{\rm p}} = 1$ and a spatial period $\lambda_{\rm phase} = \frac{2\pi}{\Omega}\frac{c^2}{w_{\rm p}} = \frac{2\pi}{\omega}c = 0.1$ which have to be compared with the temporal and spatial period of the group wave $T_{\rm group} = \frac{2\pi}{\Omega}\frac{c}{w_{\rm p}} = \frac{2\pi}{\omega} = 0.1 \leq T_{\rm phase}$, $\lambda_{\rm group} = \frac{2\pi}{\Omega}c = \frac{2\pi}{\omega}\frac{c}{v_{\rm p}} = 1 \geq \lambda_{\rm phase}$ (as illustrated in Fig. 3 for $\lambda_{\rm phase}/\lambda_{\rm group} = 0.1$. Clearly the role of phase and group waves have been swapped compared to the bradyonic case. This stresses the strong symmetry existing between our bradyonic and tachyonic particle.

Remarkably, it means that a same $u-$wave can carry several bradyons of velocity $v_{\rm p}$ associated with a spring of pulsation $\omega_{\rm p}$ and tachyons of velocity $w_{\rm p} = \frac{c^2}{v_{\rm p}}$ but with a spring of pulsation $\Omega_{\rm p}$ (compare Figs. 2 and 3). The different particles could actually move together on the same string since the transparency condition $N(t) = 0$ ensures that the particles do not interact with the wave and completely ignore each other. Therefore, we have here the possibility to generate collective excitations actually reminiscent of bosons surfing on a given wave. Consider for instance the case of n particles (bradyons or tachyons) coherently driven by a carrying $u-$wave. The situation shows some similarity with a Fock state $|n\rangle$ in quantum mechanics for a collective excitation (e.g., photons or phonons) with n energy quanta. We could imagine a statistical ensemble of such strings carrying a various number of quanta $n = 0, 1, ..., +\infty$ and subsequently develop a particle bosonic statistics with partition function $Z = \sum_{n=0}^{n=+\infty} e^{-\frac{n\varepsilon_{\omega'}}{K_B T}}$ (K_B is the Boltzmann constant and T the temperature of the ensemble). The energy $\varepsilon_{\omega'}$ is the Hamiltonian $H_{\rm p}$ associated with the bradyons or tachyons characterized by the frequency ω'. We naturally obtain Planck's law or more generally and realistically a bosonic statistics if the number of particle is finite and fixed. Clearly, this issue is very interesting for developing mechanical analogs of quantum statistics and will deserve further studies in connection with equilibrium and non-equilibrium thermodynamics.

IV. CONCLUSIONS

We have developed a simple non-relativistic mechanical analog of de Broglie's wave-particle duality. In the model considered, an oscillating bead or particle mimics an internal quantum clock in phase with a transverse acoustic wave field $u(t, x)$. Despite its non relativistic nature the model is based on a Lorentz transformation where the sound celerity c of waves on the string replaces the light velocity. Therefore the model offers strong similarities with de Broglie's approach which was based on special relativity. De Broglie named this phase-locking feature the 'phase-harmony' [3] since it emphasizes the fundamental role of the quantum relation $m_{\rm p}c^2 = \hbar\omega_{\rm p}$. Like with de Broglie's theory the acoustic $u-$field here generates a phase-wave Ψ acting as a guiding field for the particle in pure analogy with Bohmian interpretation of QM. In other words, in the presented model the u-field unifies the particle and the wave in a inseparable structure as summarized by the holonomic condition $z_{\rm p} = u(t, x_{\rm p})$. Thus, one can speak about 'wave monism' and in that limited sense our analogy deciphers the meaning of wave-particle duality. Therefore, our work enables to figure out and visualize clearly a possible mechanical interpretation of the wave particle duality and especially of the phase-harmony. It gives a deeper understanding of the possible cause for the clock synchronization on which leans de Broglie's theory.

Interestingly our approach which was developed for both the 'bradyonic' (i. e., subsonic) and 'tachyonic' (i. e., supersonic) regime is *a priori* not limited to the uniform motions. Indeed, the set of equations 10a, 10b, and 10c can easily be extended in order to include the effects of a more complex potential $V(t, x_{\rm p}, z_{\rm p})$. For example, by adding a longitudinal potential $V(t, x_{\rm p})$ Eq. 10a becomes

$$m_{\rm p}\ddot{x}_{\rm p}(t) = -\partial_x u(t,x)|_{x=x_{\rm p}(t)}\, N(t) - \partial_x V(t,x)|_{x=x_{\rm p}(t)} \quad (45)$$

whereas Eqs. 10b, and 10c are let unaffected. The model could thus in principle be applied to regimes involving confining potentials which are of particular interest for further studies on mechanical analogs of QM. In that context, the development of the numerical method employed for generating the movie (see [23]) will bring further insight into the complex dynamic of the system (especially when the 'transparency' condition $N(t) = 0$ is no longer valid). Finally, while the basis of our mechanical model is particularly simple, the complexity of the various dynamics (with $N = 0$ and $N \neq 0$) open interesting and fundamental questions. We think in particular of the unexplored tachyonic regime as well as the highly likely chaotic regimes that can also be addressed experimentally. For all these reasons, we hope that this work will stimulate further studies exploiting the potential of de Broglie's mechanical interpretation of quantum physics.

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