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A Lagrangian formulation for a gravitational analogue of the acoustic radiation force

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Abstract – In this letter, we propose an expression for the *instantaneous* acoustic radiation force acting on a compressible sphere when it is immersed in a sound field with a wavelength much larger than the particle size (Rayleigh scattering regime). By following a Lagrangian approach, we show that the leading term of the radiation force can alternatively be expressed as a fluctuating gravitation-like force. In other words, the effect of the acoustic pressure gradient is to generate a local acceleration field encompassing the sphere, which gives rise to an apparent buoyancy force, making the object move in the incoming field. When averaging over time, we recover the celebrated Gor'kov expression and emphasize that two terms appear, one local and one convective, which identify with the well-known monopolar and dipolar contributions.

Since Rayleigh's pioneering work on sound waves [1], later followed by Langevin and Brillouin, among others [2–4], it is known that like its electromagnetic cousin, acoustic waves can transfer linear momentum to a particle, even in a perfect fluid, referred to as the radiation or pressure force, with its associated tension or radiation stress tensor [4]. One might wonder how an elemental sound wave, with its harmonic pressure-velocity oscillations (in time and space), could exert a non-zero average force. Like with electromagnetic waves, the answer lies in the second-order effect arising for sound waves when the particle pulsates in volume while moving back and forth in the acoustic oscillating flow, yielding a small hysteretic displacement. These incremental displacements accumulate over millions of cycles per second (at the sound frequency), and may be interpreted as the result of an average or macroscopic force on the particle. The *average* force was first calculated by King for a hard sphere in a perfect fluid [5], and generalized by Yosioka [6] for compressible objects. It generated a complete and active field of research with increasingly heavy mathematics addressing complex objects, wave fields, and more realistic effects [7,8]. In the meantime, impressive applications of the radiation force to microfluidics have been published in the last ten years. This has given rise to the new discipline of acoustofluidics [9], in which the ability of standing waves to arrange, trap and sort particles or living cells has been demonstrated both in propagative [10] and evanescent fields [11].

However, despite great theoretical and experimental results, very few papers address the question of the short-time dynamics of a particle in a sound field, when pushed upon by the radiation force.

As is often the case for non-stationary problems [12], the use of a Lagrangian approach brings a new viewpoint on the issue. For instance, in fully developed turbulence, this renewed viewpoint brought important breakthroughs in this long-history issue [13,14]. This is the issue motivating the present work on the radiation force. Here, in order to gain insight into the physics at play, we will focus on the instantaneous dynamics of the particle when tracked in the oscillating sound flow. By leaning on the celebrated Gor'kov expression of the radiation force [15], we will show that the radiation force can also be interpreted as the average of a fluctuating buoyant force associated with an effective gravitational field, shedding new light on its physical origin.

As our paper is based on the Gor'kov formulation, we start by analyzing the implicit assumptions and the applicability of the Gor'kov formulation in the context of a free-to-move (not suspended) particle. It is to our knowledge an issue which has not been discussed in detail in the literature so far although it is mostly employed in this context.

Let us consider a compressible particle of varying outer surface $S_p(t)$ immersed in a inviscid and infinite compressible fluid, in the absence of gravity. The whole system is excited by a time-harmonic acoustic wave of frequency $f = 1/T$ characterized by its incident Eulerian velocity, pressure and density fields, respectively ($\boldsymbol{v}_{\rm in},\ p_{\rm in},\ \varrho_{\rm in}$). The acoustic wavelength λ_f in the fluid is supposed to be much larger than the sphere radius a (Rayleigh regime): $a \ll \lambda_f$. Let us choose a much larger surface S in the far-field region. $V(t)$ is the control volume V delimited externally by S and internally by $S_p(t)$.

In an inviscid fluid, only pressure forces can apply so that a particle at position $\mathbf{r} = \mathbf{r}_p(t)$ at time t is submitted to the *instantaneous* (radiation) force:

$$\mathbf{F}(\mathbf{r}_p(t)) = \int_{S_p(t)} p\mathbf{d}\boldsymbol{S}, \qquad (1)$$

where p is the *total* pressure field (incident and scattered), and the surface element $\mathbf{d}\boldsymbol{S}$ is oriented towards the particle. In general, the instantaneous force $\mathbf{F}(t)$ is not known, only its averaged value $\bar{\mathbf{F}}_{\rm rad}(\mathbf{r})$, defined as the averaged radiation force:

$$\bar{\mathbf{F}}_{\rm rad}(\mathbf{r}) = \langle \mathbf{F}(\mathbf{r}_p(t))\rangle, \qquad (2)$$

where $\langle . \rangle$ is the time-average operator over the time T.

In the general case, the momentum change rate of the fluid volume V can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \varrho\mathbf{v}\mathrm{d}V = -\int_S \mathbf{\Pi}\cdot\mathbf{d}\boldsymbol{S} - \int_{S_p(t)} p\mathbf{d}\boldsymbol{S}, \qquad (3)$$

where $\mathbf{\Pi}$ is the total momentum density flux tensor defined as $\Pi_{ij} = p\delta_{ij} + \varrho v_i v_j$. Averaging eq. (3) over time gives:

$$\begin{aligned}\bar{\mathbf{F}}_{\rm rad} &= -\left\langle \frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \varrho\mathbf{v}\mathrm{d}V \right\rangle - \int_S \langle\mathbf{\Pi}\rangle\cdot\mathbf{d}\boldsymbol{S} \qquad (4)\\ &= -\int_S \langle\mathbf{\Pi}\rangle\cdot\mathbf{d}\boldsymbol{S} - \frac{\boldsymbol{P}(T)-\boldsymbol{P}(0)}{T}, \qquad (5)\end{aligned}$$

with

$$\boldsymbol{P}(t) = \int_{V(t)} (\varrho\mathbf{v})(t)\mathrm{d}V. \qquad (6)$$

In order to simplify the problem, let us first assume that the movement of the particle mass center in the sound flow is perfectly periodic in space and time so that the particle mean displacement is exactly zero. This can be achieved by means of an additional external and time independent force, hereafter denoted $\mathbf{F}^*$. Hence, we have:

$$\mathbf{F}^* + \int_{S_p(t)} p\mathbf{n}\mathrm{d}S = m_p\mathbf{\Gamma}(t), \qquad (7)$$

with $\mathbf{\Gamma}$ and m_p the particle acceleration and mass, respectively.

By averaging over time, it is clear that the force $\mathbf{F}^*$ must exactly compensate the mean radiation force $\bar{\mathbf{F}}_{\rm rad} = \langle\int_{S_p} p\mathbf{n}\mathrm{d}S\rangle$ to ensure a zero-mean displacement (or acceleration). Thanks to the additional force, the movement is perfectly periodic in time, so that the term $\langle\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)}\varrho\mathbf{v}\mathrm{d}V\rangle$ cancels. With this assumption of a *suspended* particle, we recover the expression for the radiation force as used by Gor'kov (first equation of [15]), also in agreement with the assumption of perfect stationarity in the more detailed paper of Settnes and Bruus (see eqs. A1a to A1f of ref. [16]). In this framework, and assuming a small acoustic Mach number $\varepsilon = \frac{v_{in}}{c_{f_0}}$ (with c_{f_0} the celerity of sound in the fluid), Gor'kov has used an asymptotic approach to calculate the term $\langle\mathbf{\Pi}\rangle$. Introducing an exponent α in order to express the particle size to wavelength ratio as a function of the Mach number as done in [17], and recognizing that the leading term in $\langle\mathbf{\Pi}\rangle$ only depends on the particle monopolar and dipolar contributions, Gor'kov has shown that for a standing incident field the leading term of $\bar{\mathbf{F}}_{\rm rad}$ is $\rho c^2 a^2 O(\varepsilon^{2+\alpha})$ and derives from the acoustic potential $U(\mathbf{r})$ so that

$$\bar{\mathbf{F}}_{\rm rad} \simeq -\int_S \langle\mathbf{\Pi}\rangle\cdot\mathbf{d}\boldsymbol{S} = -\boldsymbol{\nabla}U(\mathbf{r}), \qquad (8)$$

$$\text{with}\quad U(\mathbf{r}) = V_{p0}\left(\frac{f_1}{2}\kappa_{f0}\langle p_{\rm in}^2\rangle - \frac{3f_2}{4}\varrho_{f0}\langle v_{\rm in}^2\rangle\right) \qquad (9)$$

$$\text{and}\quad f_1 = 1-\tilde{\kappa}, \quad\text{and}\quad f_2 = \frac{2(\tilde{\varrho}-1)}{2\tilde{\varrho}+1}, \qquad (10)$$

$\tilde{\kappa} = \frac{\kappa_{p0}}{\kappa_{f0}}$ and $\tilde{\varrho} = \frac{\varrho_{p0}}{\varrho_{f0}}$ being, respectively, the equilibrium compressibility and density ratios of the particle over the fluid, while V_{p0} is the particle rest volume.

Now, in the more general case of a free particle (*i.e.*, when $\mathbf{F}^* = \mathbf{0}$) the particle movement is no longer exactly time-periodic so that a tiny incremental displacement $\delta(t)$ is added at every sound cycle. Consequently, the fluid momentum is no longer periodic and the momentum term $\langle\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)}\varrho\mathbf{v}\mathrm{d}V\rangle$ is *a priori* not exactly zero and the total radiation force differs from the Gor'kov expression. In fact, as we show in the Supplemental Material `Supplementarymaterial.pdf` (SM) (see SM1), the difference between the zero-mean displacement particle and the free-to-move particle yields a correction in terms of forces which is of higher order. Consequently, estimating the force in a Lagrangian approach over the time symmetric trajectory corresponding to a suspended particle is as good, in terms of order of approximation, as relying on the Gor'kov formulation. By using an asymptotic approach detailed below, we are going to derive the expression for an equivalent instantaneous radiation force $\mathbf{F}_{\rm rad}(t)$, so that at the leading order in the Mach number, $\mathbf{F}(t) = \mathbf{F}_{\rm rad}(t) + \mathbf{f}(t)$, with $\mathbf{f}$ a zero-mean function.

Let us first define the scattered pressure field p_s as the correction of the incident field required to account for the presence of the particle:

$$p = p_{\rm in} + p_s. \qquad (11)$$

From eqs. (1) and (2) it is thus always possible to calculate $\bar{\mathbf{F}}_{\rm rad}$ as $\bar{\mathbf{F}}_{\rm rad}(\mathbf{r}) = \langle \int_{S_p(t)} (p_{\rm in} + p_s)\mathbf{d}\boldsymbol{S}\rangle$. This was the Eulerian approach used in the seminal work of Yosioka [6] in which an expansion equivalent to the one used in optics by Mie for scattering of light by spherical particle was done to deduce eq. (8).

Here, we instead consider a Lagrangian description of the particle in movement in the surrounding fluid. For this purpose, we first make a guess that the motion of the particle is mainly driven by the effect of $p_{\rm in}$. As a first approximation we could be tempted to write $\bar{\mathbf{F}}_{\rm rad}(t) = \langle \int_{S_{p,in}(t)} p_{\rm in}\mathbf{d}\boldsymbol{S}\rangle$ where $S_{p,in}(t)$ is the surface of the particle altered only by the effect of $p_{\rm in}$ (see SM1 in the SM for a rigorous definition). However, this approximation is in general too crude. Indeed, in the limit case of a fluid particle in fluid, *i.e.*, with $f_1 = f_2 = 0$, we get from eqs. (8) and (10) that $\bar{\mathbf{F}}_{\rm rad} = \mathbf{0}$. This contradicts the elemental fact that, in such a case, for a standing wave, $\langle \int_{S_{p,in}(t)} p_{\rm in}\mathbf{d}\boldsymbol{S}\rangle$ doesn't actually vanish (see SM3 in the SM). Therefore, the expression should be corrected, the simplest one we have found being of the form $\bar{\mathbf{F}}_{\rm rad} \simeq \langle \mathbf{F}_a(t)\rangle$ with

$$\mathbf{F}_a(t) = -\beta(t)\int_{S_{p,in}(t)} p_{\rm in}\mathbf{d}\boldsymbol{S}, \tag{12}$$

where the specific correction $\beta(t) = \frac{\varrho_p - \varrho_f}{\varrho_{f0}}$ depends on $\varrho_p(t)$ and $\varrho_f(t)$, which are the instantaneous particle and fluid densities in the mere incident field, respectively. $\mathbf{F}_a(t)$ has the peculiar property to cancel at all times for a neutral particle (*i.e.*, when no radiation force is present). Remarkably, we will show that eq. (12) allows us to recover Gor'kov results, *i.e.*, eq. (8).

Using the divergence theorem together with the small particle assumption, eq. (12) yields at the leading order:

$$\mathbf{F}_a(t) = \frac{(\varrho_p - \varrho_f)}{\varrho_{f0}}\boldsymbol{\nabla} p_{\rm in}(\mathbf{r}_p(t))V_{p,in}(t). \tag{13}$$

Then, in order to calculate $\langle \mathbf{F}_a(t)\rangle$, one must remember that the particle constantly moves in the field. To simplify the present calculations, we can define $\tilde{\mathbf{F}}_a(\mathbf{r}, t)$, obtained from the expression of $\mathbf{F}_a(t)$ assuming the particle stands at $\mathbf{r}$ instead of $\mathbf{r}_p(t)$ at time t, thus satisfying

$$\mathbf{F}_a(t) = \tilde{\mathbf{F}}_a(\mathbf{r}_p(t), t). \tag{14}$$

Defining the initial time $t = 0$ by $\mathbf{r}_p(\mathbf{0}) = \mathbf{r}$, we then have

$$\mathbf{F}_a(t) \simeq \tilde{\mathbf{F}}_a(\mathbf{r}, t) + \Big(\mathbf{r}_p(t) - \mathbf{r}_p(0)\Big)\cdot\boldsymbol{\nabla}\tilde{\mathbf{F}}_a(\mathbf{r}, t). \tag{15}$$

We identify both a *local* and *convective* contributions, respectively denoted $\mathbf{F}_a^{\rm loc}(t)$ and $\mathbf{F}_a^{\rm conv}(t)$, so that

$$\mathbf{F}_a(t) \simeq \mathbf{F}_a^{\rm loc}(t) + \mathbf{F}_a^{\rm conv}(t), \tag{16}$$

with

$$\mathbf{F}_a^{\rm loc}(t) = \tilde{\mathbf{F}}_a(\mathbf{r}, t), \tag{17}$$

and

$$\mathbf{F}_a^{\rm conv}(t) = \int_0^t \boldsymbol{v}_p(t')\mathrm{d}t'\cdot\boldsymbol{\nabla}\tilde{\mathbf{F}}_a(\mathbf{r}, t). \tag{18}$$

The local force can be expressed, keeping terms up to order $O(\varepsilon^{2+\alpha})$, as

$$\mathbf{F}_a^{\rm loc}(t) = -V_{p0}\left[-\frac{3}{2}\frac{f_2}{1-f_2}\boldsymbol{\nabla} p_{\rm in} + \kappa_{f0}\frac{f_1}{2}\boldsymbol{\nabla} p_{\rm in}^2\right](\mathbf{r}, t), \tag{19}$$

where use of Euler's equation has been made as detailed in the SM.

Averaging the above expression over time yields

$$\langle \mathbf{F}_a^{\rm loc}(\boldsymbol{r_p}(t))\rangle = -\boldsymbol{\nabla}\left[V_{p0}\kappa_{f0}\frac{f_1}{2}\langle p_{\rm in}^2\rangle\right]. \tag{20}$$

It is noteworthy that the term in brackets corresponds to the first term (monopolar contribution) of the Gor'kov force potential expressed in eq. (9).

In order to obtain the mean convective term contribution, we start expressing the first order particle velocity $\boldsymbol{v}_p(t)$ as a function of the incident acoustic velocity field $\boldsymbol{v}_{\rm in}$. The particle being accelerated in the incident sound field of velocity $\boldsymbol{v}_{\rm in}(\mathbf{r}, t)$, the surrounding fluid inertia leads to an added mass effect (see [18]), which first appears at the order $O(\varepsilon)$: At this order, Batchelor [19] shows that $\boldsymbol{v}_p$ can be expressed as

$$\dot{\boldsymbol{r}}_p(t) = \boldsymbol{v}_p(t) = (1 - f_2)\boldsymbol{v}_{\rm in}(\boldsymbol{r}_p(t)). \tag{21}$$

This equation represents the zero-mean trajectory followed by a dense particle in a sound field (at order 1 in Mach). Now, inserting expressions (19) and (21) in eq. (18), we get

$$\mathbf{F}_a^{\rm conv}(t) = V_{p_0}\frac{3}{2}f_2\int_0^t \boldsymbol{v}_{\rm in}(\boldsymbol{r}, t')\mathrm{d}t'\cdot\boldsymbol{\nabla}\left(\boldsymbol{\nabla} p_{\rm in}(\boldsymbol{r}, t)\right). \tag{22}$$

After some calculations detailed in SM3 in the SM, involving the linearized Euler equation and an integration by parts, it follows that

$$\langle \mathbf{F}_a^{\rm conv}(\mathbf{r}_p(t))\rangle = -\boldsymbol{\nabla}\left[-V_{p0}\frac{3f_2}{4}\varrho_{f0}\langle v_{\rm in}^2\rangle\right]. \tag{23}$$

Likewise, the term in brackets is the dipolar contribution of the Gor'kov potential expressed in eq. (9).

Combining the mean local and convective contributions given in eqs. (20) and (23), we obtain, at the leading order,

$$\bar{\mathbf{F}}_{\rm rad} \simeq \langle \mathbf{F}_a(\mathbf{r}_p(t))\rangle. \tag{24}$$

$\mathbf{F}_a(t)$ can thus be identified to the instantaneous radiation force $\mathbf{F}_{\rm rad}(t)$ defined in eq. (8) in complete agreement with Gor'kov's results. This Lagrangian derivation of the radiation force constitutes the first main finding of this letter.

The second important result, that we will now discuss, concerns the physical interpretation of this radiation force.

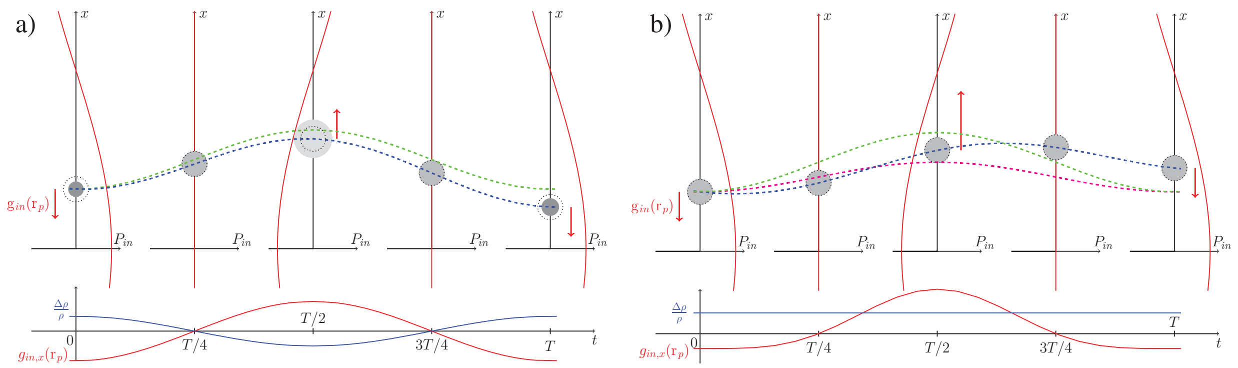

Fig. 1: Time sequence of a particle over one period T of a standing wave propagating along the (vertical) x-axis. It illustrates the radiation force as a gravity-like effect for two representative cases plotted according to eq. (25): (a) a compressible and neutrally buoyant particle and (b) a dense and iso-compressible particle. The green dashed line is the trajectory the particle would follow as a fluid particle, and the blue dashed line is the actual one. In case (b) is also added in magenta the trajectory the particle would have without the added-mass effect. The sphere volume is filled with grey color, its equilibrium shape being delimited with the black dotted line. As it plays no role in the second case, we chose to represent an incompressible particle. Below each sequence, both the local relative density shift $\Delta\varrho/\varrho$ and the acoustic gravity component $g_{in,x}$ are plotted to aid the interpretation. See the text for a more detailed step-by-step explanation.

For this purpose we introduce an effective gravitation field $\boldsymbol{g}_{\text{in}} = \frac{\boldsymbol{\nabla} p_{\text{in}}}{\varrho_{f0}}$ and a relative density $\Delta\varrho = \varrho_p - \varrho_f$ in eq. (13) and eq. (24), which leads to

$$\bar{\mathbf{F}}_{\text{rad}} = \langle \Delta\varrho V_p \boldsymbol{g}_{\text{in}} \rangle. \tag{25}$$

The radiation force can thus be understood as the leading term in the mean force which would result from a gravitational field modulation equal to $\boldsymbol{g}_{\text{in}}(t)$. In other words, $\mathbf{F}_a(t)$ can be seen as a *fluctuating* apparent weight, resulting from the combination of two oscillating quantities:

- a forcing effect: an incident acoustic gravity-like acceleration field $\boldsymbol{g}_{\text{in}}(\mathbf{r}, t)$;
- a response effect: owing to their compressibility, both the particle volume and the fluid densities oscillate at the forcing frequency, rendered by the relative density term $\Delta\varrho V_p = \frac{\Delta\varrho}{\varrho_p} m_p$ (m_p: constant particle mass).

By analogy with the buoyancy force (*i.e.*, Archimedes' law), arising when a particle has a density or compressibility different from the fluid in which it is immersed, the oscillating force $\mathbf{F}_a(t)$ is equivalent to a rapidly fluctuating "acoustic gravitational force". As we have shown, the time-average of this force, taking both the temporal and spatial structure of the field into account, leads to the classic radiation force expressed by Gor'kov for standing waves. Our conclusion is also in agreement with Gor'kov's work for progressive wave for which the radiation force is expected to be zero at this order as well.

We will now give a comprehensive picture of the physics at play in the gravitation-like force, considering a particle in a plane standing wave $p_{\text{in}} = -p_0 \cos(\omega t) \cos(kx)$, with $k = 2\pi/\lambda$ and $\omega = kc_{f0}$ the wave number and angular frequency of the wave, respectively. As explained above, the acoustic gravitational effect results from two contributions: a local one, associated with the oscillation of the particle apparent mass $m_p(1 - \frac{\varrho_f}{\varrho_p})$ in the acoustic gravitation field, and a convective one, linked to the local exploration of the field by the oscillating particle. Let us now separate both contributions by considering two limit cases for a particle initially located between a pressure antinode (at $x = 0$) and the nearest node in the $x > 0$ region (see fig. 1).

Case a: *a neutrally buoyant but compressible particle.* We first consider a particle both neutrally buoyant ($\varrho_{p0} = \varrho_{f0}$) and more compressible than the fluid ($\kappa_{p0} > \kappa_{f0}$, *i.e.*, $f_1 < 0$). In this case, only the local contribution remains as the convective term vanishes ($f_2 = 0$). Let us figure out the particle movement over a time period T. The sinusoidal green dashed line in fig. 1(a) represents the movement of a *fluid* particle in the sound wave, which is also, at leading order, the particle movement since $f_2 = 0$ so that $\boldsymbol{v}_p = (1 - f_2)\boldsymbol{v}_{\text{in}} = \boldsymbol{v}_{\text{in}}$ (no added mass effect comes into play in this case).

At time $t = 0$, the pressure gradient is such that $\boldsymbol{g}_{\text{in}}$ is oriented downward[1]. The particle is compressed and hence denser than the hosting fluid at its location, so that it plunges downwards. At $t = T/4$, the pressure is zero, the particle thus follows the non perturbed trajectory (no radiation force). At $t = T/2$, the pressure gravity reverses but the particle also expands, so that it is 'lighter' and thus keeps sinking. At $t = 3T/4$ it follows the same trajectory (no force). Overall, the acoustic gravity is always out of

[1]For the sake of conciseness, we will abusively use the terms up (towards $x > 0$) and down (towards $x < 0$), assuming the classical paper reading orientation on the Earth.

phase with the density shift: for the whole period, the particle keeps "falling" towards the pressure antinode at $x = 0$ and the instantaneous radiation force maintains the same orientation.

Case b: *an iso-compressible particle, denser than the fluid.* We now consider a particle iso-compressible ($f_1 = 0$) but denser than the hosting fluid ($f_2 > 0$), also located between $x = 0$ and $x = \pi/k$, as shown in fig. 1(b). To the green fluid particle trajectory is now added the flattened magenta dashed line, which represents the trajectory the particle would follow if only added-mass effect would apply. First, the particle dynamic is such that it plunges between $t = 0$ and $t = T/4$ since it is denser than the fluid in a downward gravity field. However, at $t = T/2$, the gravity field reverses so that the particle rises. Here, the point is that the gravity field (or acoustic pressure gradient) being larger at the location where the particle is at $T/2$ than at $t = 0$, the radiation force over the cycle does not balance symmetrically. On average, a net upward radiation force (toward the nearest node) is exerted upon the particle.

It is noteworthy that in this case, the effect of the added mass is to alter (with a factor $(1 - f_2)$) the particle trajectory obtained by integrating $\boldsymbol{v}_p = (1 - f_2)\boldsymbol{v}_{\text{in}}$. In other words, the "landscape" explored by the particle determines the hysteretic values of the gravity field that the particle will encounter at extremal positions and henceforth contributes to its amplitude. In the convective (or 'landscape') effect, we note that the radiation force reverts within a cycle while it is not the case in the local one, where it maintains the same orientation.

In conclusion, the time-resolved Lagrangian approach turns out to be another way to interpret both the time-averaged scattered contributions (monopolar and dipolar scattering terms) appearing in the Gor'kov potential of the averaged radiation force. Equation (25) gives a clear interpretation (see fig. 1) of the acoustically induced buoyancy effect, from which both terms are derived: i) a local term, originating from the compressibility ratio, a neutrally buoyant particle alternatively sinking and floating within the time varying acoustic field, and ii) a convective one, a denser but iso-compressible particle encountering a stronger instantaneous force as it approaches pressure nodes where the pressure gradient is the largest. Overall, it shows that the radiation force has the structure of an inertial force in an *equivalent* gravity acceleration field created by the acoustic pressure gradients in the fluid. Therefore, it is possible to consider the acoustic radiation force as a "centrifugal" force. This idea is indeed reinforced by the ability of the acoustic radiation force to sort particles according to their size, density and compressibility ratios (acoustophoresis [9]), which is reminiscent of centrifugal forces. Correlatively, our findings may also shed new light on the ability of the radiation force to universally deform interfaces [20], or to separate miscible fluids of different densities, as evidenced by different groups [21–23].

As a perspective, we hope our work will stimulate future experimental investigations in the direction of ultrafast tracking of the time resolved particle's dynamic in a sound field.

Beyond acoustics, one may wonder how this approach could be transposed to other types of waves such as electromagnetic waves, transverse waves on a string or surface waves (*e.g.*, in hydrodynamic quantum analogs involving walking droplets bouncing on a vibrating bath [24]) for which a radiation force also arises. Finally, our work reveals the emergence of a gravitational field from a wavy background pushing an object (thus flowing in a quiescent fluid). For that reason, this result could have been expected from the point of view of acoustical analogs of blackholes [25] for which it is known that an acoustical field superimposed on a *flowing* fluid yields the curved space-time metric of the general relativity [26]. Hence, and more prospectively, the similarity of both issues suggests for the future to address the problem of acoustical analogs with a point of view centered around the radiation force phenomenon.

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